Exercise 61

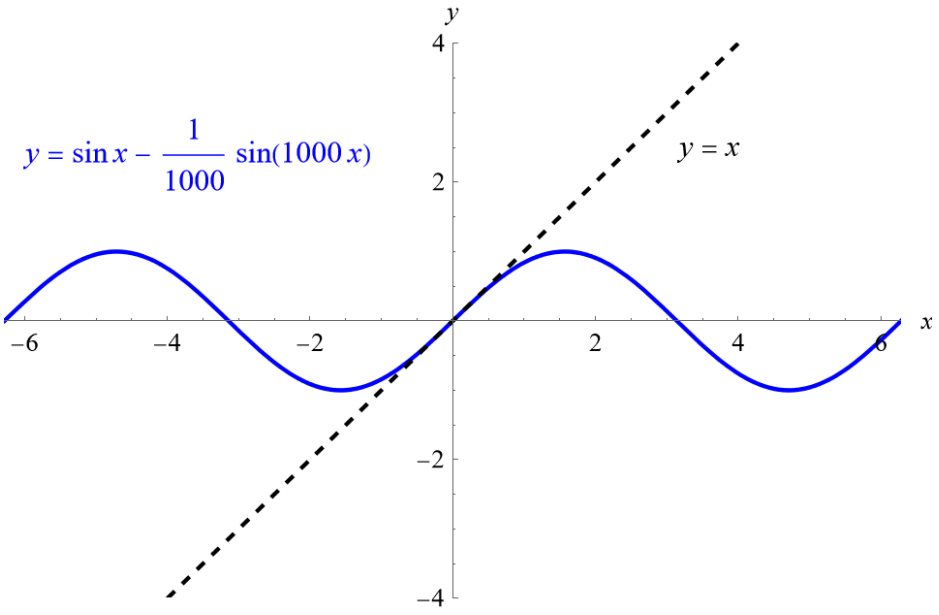
(a) Graph the function $f(x) = \sin x - \frac{1}{1000}\sin(1000x)$ in the viewing rectangle $[-2\pi, 2\pi]$ by $[-4, 4]$. What slope does the graph appear to have at the origin?

(b) Zoom in to the viewing window $[-0.4, 0.4]$ by $[-0.25, 0.25]$ and estimate the value of $f'(0)$. Does this agree with your answer from part (a)?

(c) Now zoom in to the viewing window $[-0.008, 0.008]$ by $[-0.005, 0.005]$. Do you wish to revise your estimate for $f'(0)$?

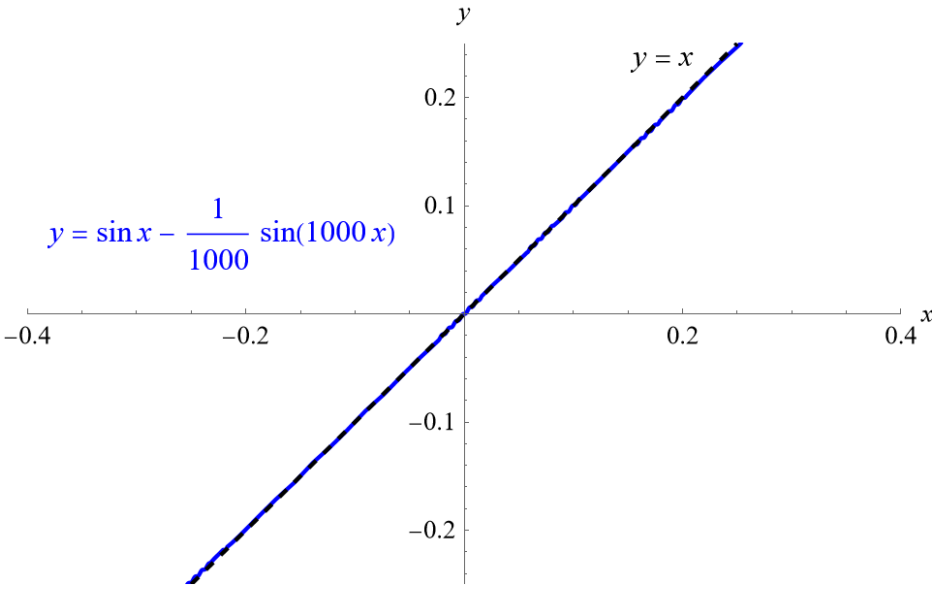
Solution

Below is the graph of $f(x)$ versus x in the viewing rectangle $[-2\pi, 2\pi]$ by $[-4, 4]$.

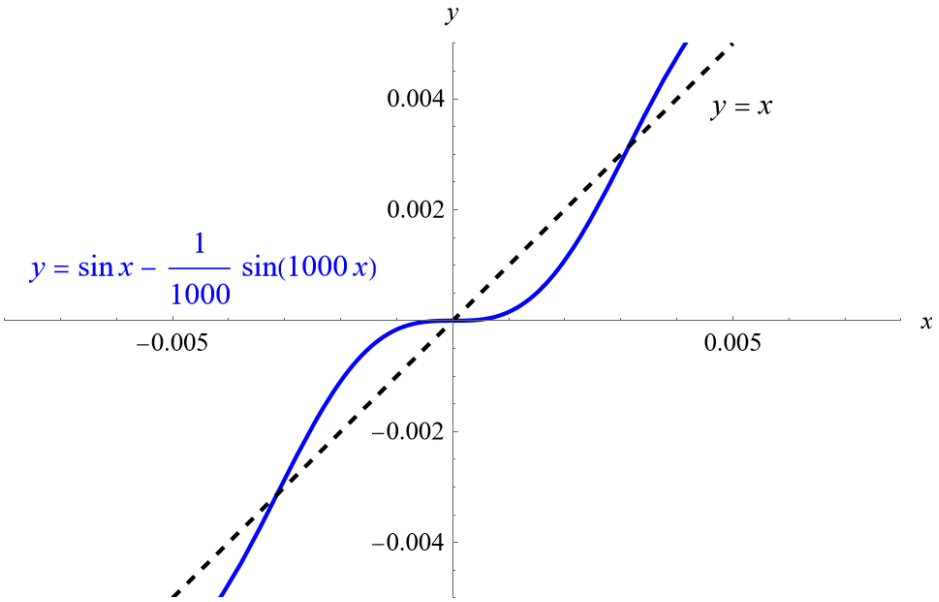


The slope of the graph at the origin appears to be 1: $f'(0) = 1$.

Below is the graph of $f(x)$ versus x in the viewing rectangle $[-0.4, 0.4]$ by $[-0.25, 0.25]$.



The slope of the graph at the origin still appears to be 1: $f'(0) = 1$. Below is the graph of $f(x)$ versus x in the viewing rectangle $[-0.008, 0.008]$ by $[-0.005, 0.005]$.



The slope of the graph at the origin is actually zero: $f'(0) = 0$.